Selected Writing Behaviors of Fifth Graders As They Composed Original Mathematics Story Problems

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This five-month study examined the writing behavior of 8 fifth-grade students as they composed original mathematics story problems. An analysis of observations indicated that students developed three strategies for writing story problems. First, students usually engaged in question-directed behavior during problem writing. After they identified the general problem topic, students generated the culminating question that they then used to guide composition of the problem text. At other times, students used a free association strategy: They generated their information by making associations with the general topic, wrote the problem text, and then identified the culminating question. Finally, all students developed techniques to make their problems more difficult. Overall, the analysis of students, problem writing behavior contributes to an understanding of the story-problem writing process. The study further suggests that students may be able to collaborate effectively with teachers in writing the mathematics curriculum.

Educational theorists (Dewey, 1910; Getzels, 1964) and practitioners (Brown & Walter, 1983; Graves, 1983) have argued that students should have regular opportunities to pose their own questions and problems. Rationales for a problem-posing curriculum vary from an argument that problem posing is the first step of the problem-solving process (Dewey, 1910; Getzels, 1964) to the claim that problem ownership results in improved student performance on problem-solving activity (Graves, 1975), to the idea that problem posing is typical of practitioners in the disciplines, that is, in mathematics (Brown & Walter, 1983) and in science.

However, pedagogical strategies that lead students to deliberately and systematically pose their own problems are unusual, particularly in the traditional academic disciplines. With respect to the elementary mathematics curriculum, there is little in the research or the teaching literature that supports students' composition of mathematics story problems (cf., however, Fennell & Ammon, 1985; Ford, 1990; Kilpatrick, 1987). But there has been increasing interest in the uses of writing to support learning in mathematics (Azzolino, 1990; Countryman, 1992; Evans, 1984; National Council of Teachers of Mathematics, 1989). Many writing-to-learn mathe-
matics activities, for example, entail the use of journals in which students reflect upon some problem-solving activity (e.g., Gordon & MacInnis, 1993). Still, for the most part, the mathematics problems that students are asked to solve are written by adults.

The focus of this article is part of a larger study that examined students' responses when they were invited to compose their own mathematics story problems (Winograd, 1991a). For five months in one fifth grade classroom, I observed students as they wrote, solved, and shared these problems with peers. This paper reports several of the writing behaviors of students as they composed original mathematics story problems. I wanted to understand the behavior of these students as they wrote problems, particularly when instruction on problem writing was unstructured and open-ended. In the teaching literature, there have been periodic calls to have students write their own mathematics story problems (e.g., Larkin, 1985). However, except for several brief suggestions that urge mathematics problem writers to use a writing-process approach (Fennell & Ammon, 1985; Ford, 1990), a more developed theory that would help teachers support students' development as mathematics story-problem writers does not exist.

Theoretical Background

A Study of Problem-Writing Behavior: What's the Point?

A study of students' composition of mathematics story problems makes sense for several reasons. First, mathematics education remains heavily discipline- and text-centered in contrast with a large segment of the reading and writing education community wherein students have been given more control over curriculum (e.g., Harste, Short, & Burke, 1988). My interest was to develop a practical model for teaching elementary mathematics that had a student-centered orientation similar to that in the language arts. Second, problem posing is said to be a basic activity of mathematicians that reflects mathematics as a creative, generative activity (Brown & Walter, 1983; Kilpatrick, 1987). By inviting students to write their own mathematics problems, a more authentic version of mathematics' practice may be conceptualized for the school curriculum. Finally, there is an apparent chronic malaise in American students' performance in mathematics problem solving as described by classroom teachers (e.g., Holt, 1964) and national (e.g., Koubal, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988) and international assessments (Stevenson, 1990). I was curious, therefore, to find out if a student-centered, problem-writing approach to school mathematics would have a positive effect on students' learning of mathematics problem solving: that is, improve student atti-
tudes and beliefs and their subsequent performance on problem-solving assessments. The scant research on this subject suggests that the opportunity to write problems does promote more effective problem-solving performance with elementary (Keil, 1965; Winograd, 1991a; Wirtz & Kahn, 1982), secondary (Bell & Bell, 1985), and adult learners of mathematics (Gage, 1984).

However, the present research reports a study of problem-writing behavior alone and does not examine the effects of problem writing on students' subsequent problem-solving performance. Before the effects of problem writing on problem-solving performance can be studied meaningfully, a theory or model of the mathematics problem-writing process must be formulated. Without such a theory, studies that examine the relationship between problem writing and problem solving are prone to problem-writing "treatments" that are inadequately conceptualized, superficial, or at worst, arc uninteresting to students. Research that links problem-writing and problem-solving performance should reflect well-grounded ideas about the story-problem writing process. The present research aims to propose such ideas.

**Writing as Problem Finding**

Flower and Hayes (1980) conceptualized writing as a problem-solving process in which problem finding is an integral part. When planning, the writer engages in what Flower and Hayes considered to be a crucial part of the writing process, "the act of finding or defining the problem to 'be solved'" (p. 22). It is through planning that the writing problem becomes defined, essentially in the form of goals. Flower and Hayes argued against the notion that the "rhetorical problem" is derived intuitively or simply through an act of discovery. Instead, the writing problem results from rational, goal-directed activity in which the writer actively works to create the problem that is to be solved.

Flower and Hayes considered goal-directedness to be an important characteristic that distinguishes good from poor writers. For example, good writers attend to many aspects of the rhetorical problem. During all phases of composition, good writers work to construct representations of not only the assignment and the audience but also of their own goals regarding their intended meaning, the reader, and the constraints of the genre. Good writers build "rich networks" of goals for affecting the reader which, in turn, help the writers themselves develop new ideas. Poor writers, on the other hand, are not goal-directed during composition. They are concerned mostly with the superficial features of text (e.g., length or general format), and most of their content is tied directly to the topic and not to any higher level goals.
Elementary students usually do not exhibit the planning behavior that is characteristic of more mature writers, particularly when faced with school-sponsored writing tasks (cf., Emig, 1969). Bereiter and Scardamalia (1982) and others (Burtis, Bereiter, Scardamalia, & Tetroe, 1983; Scardamalia & Bereiter, 1985) have corroborated Flower and Hayes' (1980) findings regarding the differences in planning and goal-directed behavior between expert and novice writers in their studies of elementary-aged children. Scardamalia and Bereiter described two variations of goal-directed writing behavior: the “high road” and the “low road.” The high road, typical of more mature writers, involves a writing process characterized by recursive, back and forth behavior during which time the writer continually compares his or her goals with the text as it gradually emerges. The low road, typical of less mature writers, is based on avoiding goal constraints. Whatever outcome the writing takes becomes acceptable to its author as long as it relates to the general topic. The low road approach is entirely forward-moving. This model of the more effective writer (directed by goals beyond the actual text) and the ineffective writer (directed by the writing topic and not guided by higher level goals) provides an heuristic useful in making sense of students’ mathematics story-problem writing behavior.

The Present Study

The goal of this research was to generate questions and suggest theory in one neglected domain of the elementary curriculum: mathematics problem writing. As I began this research, my question was exploratory and open ended: How do elementary students respond when they are invited to write their own mathematics story problems? The analysis of students’ problem-writing behavior will contribute to some preliminary understandings of the story-problem writing process. My assertions about the problem-writing process and the classroom conditions that might promote students’ growth as problem writers pertain directly to the 8 students in this study. For other students and settings, this study can provide a theoretical and practical point of departure for a problem-writing approach to school mathematics.

Method

Site, Participating Teacher, and Class

The site was a self-contained, fifth-grade classroom in a suburban public school located in Greeley, Colorado. The 25 students in the class were
predominantly middle class. The teacher, Kathy Phillips (a pseudonym), had taught for 23 years at the time of the study.

Instructional Setting

Students wrote, solved, and shared math story problems three or four days a week during the five-month study. Students also worked non-sequentially through the mathematics textbook (Eicholz, O'Daffer, & Fleenor, 1985). There was no conscious effort to integrate the content of the mathematics textbook and the mathematics that students drew upon in writing their story problems.

The mathematics period (every day from 1:15 PM to 2:30 PM) usually began with "mathematician's chair," a modification of "author's chair" (Graves & Hansen, 1983). In mathematician's chair, the student copied a self-generated problem on the blackboard and then directed the entire class to "take a few minutes and see if you can do my problem." What happened at this point included the following: Students called out answers; problem solvers asked for explanation; either the problem writer or a volunteer went to the board to lead explanation; other problem solvers offered alternative explanations; and the teacher intervened to address some problem-solving strategy or concept pertinent to the problem. After some consensus was reached regarding the answer, the problem writer asked, "What did you like about my problem? How can I change the problem?" Thereafter, students responded for a few minutes.

After mathematician's chair, Kathy divided the class into two groups. One group received instruction from the mathematics textbook (Eicholz et al., 1985), and the other group remained at their seats where they wrote and solved story problems and then posed these problems to individual peers. Students had complete control over the mathematics and non-mathematics content of their problems. The two groups switched activities after approximately 30 minutes. Then for 15 minutes, within groups of three, students took turns posing their problem to the other two group members. Finally, for approximately ten minutes at the end of the period, Kathy asked selected students to report to the whole class about what had transpired during small-group activity.

The instructional focus of Kathy's mathematics program was mathematics concepts and problem-solving strategies. This focus reflected the expectations of the larger mathematics education community as well as the students, parents, Kathy's colleagues, and the school district. So, instruction in problem writing was incidental and consisted primarily in immersing students in examples of one another's problems. (Students, beginning very early in the project, almost always wrote coherent problems containing mathematics that reflected some curriculum goal. There-
fore, neither Kathy nor I felt compelled to spend much time on problem-writing instruction.) Beyond this incidental instruction, students were encouraged to use topics from everyday experience in their story problems. Kathy and I led mathematician’s chair (using our own problems) approximately once a week during the first six weeks of school. Topics were chosen to carefully reflect some actual experience, although the questions were more hypothetical and speculative. A problem typical of those shared with the students early in the project follows.

Melinda and I took a three-week car trip to California. Our average daily cost was $63. We came home with $200 left. How much did we spend on the trip?

The question is hypothetical as it did not serve to solve an actual problem.

Participants

I studied the writing behaviors of 8 students, combining maximum variation and convenience strategies (Patton, 1987). The criteria for selecting students reflect characteristics of effective and ineffective problem solvers in school mathematics (e.g., Flexer, 1987; Lester & Garofalo, 1987). Students were categorized as either high status, medium status, or low status. Selection of students was based on my observations of students during problem-solving activities as well as on Kathy’s recommendations. High status students (Jessica, Aaron, Rachel, Bruce) were described as successful in school mathematics by their previous teachers; exhibited a positive mathematics self-concept; showed persistence during problem-solving activity; were interested in the meaning of problems; and were not satisfied only with knowing the correct answer. Low status students (Eli, Beth, Robbie) were described as unsuccessful in school mathematics by their previous teachers; exhibited a negative mathematics self-concept; often demonstrated “premature closure” when they became frustrated during problem-solving activity; and tended to abandon problems once the correct answer became available to them (although they may not have understood the meaning of those problems). The medium status student (Jeff), who showed characteristics of both low and high status groups, tended to behave more effectively on problem-solving tasks than the low status students but not as effectively as the high status students.

Data Collection Procedures

I audiotaped students as they engaged in think-alouds during problem writing. I used a hybrid of the clinical interview and the think-aloud procedure, referred to by Ginsburg, Kossan, Schwartz, and Swanson
(1983) as "mixed cases." Students were asked to say aloud everything that came to mind while writing problems. I occasionally questioned them during problem writing to clarify behaviors that were confusing to me. At the end of episodes, I continued questioning students about their rationales for particular behaviors. I supplemented audio-recording with written field notes of students' non-verbal gestures as well as their verbal utterances that occurred out of range of the tape recorder. Each episode was transcribed verbatim into a typed transcript.

Data Analysis

Twenty-three audiotaped problem-writing episodes were transcribed and analyzed. I analyzed in their entirety all episodes that I had observed from the student's initial consideration of a problem topic to the completion of writing.

Analysis of story-problem writing behavior was guided by Flower and Hayes' (1980) model of problem writing as a deliberative process in which the writer uses self-constructed goals that guide composition. During the later stages of data collection, I noticed that students sometimes verbalized the culminating question of their story problems early in the problem-writing process, while at other times they did not generate the culminating question until the problem text was written. (The culminating question is the question usually located at the end of a mathematics story problem.) It became apparent to me that the use of the final culminating question to guide story-problem writing was a commonly used strategy by some of the students. I therefore reread the transcripts of problem-writing episodes and coded the data guided by the questions, "At what point during the writing process do students identify the culminating question? And how do they use this question to guide story problem writing?" I then compared students' use of culminating questions with the status level of students and story problem topics.

Early in the research, I also noticed that students deliberately tried to write more difficult problems for their peers. During data analysis, I reread transcripts in search of student verbalizations that indicated their effort to write more difficult problems. I used a procedure recommended by Lincoln and Guba (1985) to categorize these strategies, designating this category, "strategies to increase problem difficulty."

Results

Two types of behavior were identified in the writing activity of the 8 students over 23 writing episodes: 1) use of culminating questions and 2) strategies to increase problem difficulty.
Use of Culminating Questions

Students tended to identify and use culminating questions early in the writing process to guide their composition of story problems. In 57% of the episodes (13 out of 23), students developed a culminating question either before or during the composition of the problem text; in 26% of the episodes (6 out of 23), students did not generate their culminating question until after the problem text was written; and in 17% of the episodes (4 out of 23), the data were insufficient to make a determination.

Five of the 6 story problems in which the culminating question was generated at the end of problem writing were written by low-status students. Except for one episode, all the middle- status and high-status students used culminating questions to guide their writing (see Table 1).

Writing Behaviors That Focused on the Culminating Questions

In 57% of the episodes, students first identified a general topic and then generated a culminating question that was related to the general topic. With the culminating question in mind, students then generated and supplied content that provided the information (i.e., problem text) necessary to answer that question. Composition of the problem text was goal-

Table 1
Number of Problems Studied and Point of Introduction of Culminating Question

<table>
<thead>
<tr>
<th>Name (status)</th>
<th>Total Number of Problems Studied</th>
<th>Point of Introduction of Culminating Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eli (low)</td>
<td>4</td>
<td>before: 0, during: 0, after: 1, unclear: 3</td>
</tr>
<tr>
<td>Beth (low)</td>
<td>3</td>
<td>before: 0, during: 0, after: 3, unclear: 0</td>
</tr>
<tr>
<td>Robbie (low)</td>
<td>2</td>
<td>before: 1, during: 0, after: 1, unclear: 0</td>
</tr>
<tr>
<td>Jeff (middle)</td>
<td>3</td>
<td>before: 2, during: 1, after: 0, unclear: 0</td>
</tr>
<tr>
<td>Jessica (high)</td>
<td>2</td>
<td>before: 1, during: 0, after: 0, unclear: 1</td>
</tr>
<tr>
<td>Aaron (high)</td>
<td>1</td>
<td>before: 1, during: 0, after: 0, unclear: 0</td>
</tr>
<tr>
<td>Rachel (high)</td>
<td>1</td>
<td>before: 0, during: 1, after: 0, unclear: 0</td>
</tr>
<tr>
<td>Bruce (high)</td>
<td>7</td>
<td>before: 4, during: 2, after: 1, unclear: 0</td>
</tr>
<tr>
<td>Totals</td>
<td>23</td>
<td>before: 9, during: 4, after: 6, unclear: 4</td>
</tr>
</tbody>
</table>
Directed in the sense that students' writing behavior was directed toward answering the final question. The process of story-problem writing in this situation was more or less recursive: Students, during problem writing, moved back and forth between the emerging written text and their idea of the culminating question.

Rachel's Problem. Rachel's bus stop problem is an example of how students used culminating questions to guide writing activity (see Figure 1; original and transcribed copies of students' problems have not been edited for spelling).

Before putting pencil to paper, Rachel established her topic.

Rachel: I'm thinking—what will I write about . . . Well, I could write you—how would someone catch up to me. That's what I was thinking about. Or, how long the bus takes to get to the stop. Or, how long it takes me to walk home.

With these questions in mind, Rachel began writing information related to her first idea, about catching up with her friend [italics added to indicate writing]: If she were walking 7 steps a minute . . . and I was walking 12 steps a minute. After some more exploration of time and rate ideas, she became frustrated.

Rachel: . . . (laughs) I don't know what to have for a question.

Researcher: What are you thinking about now?

Rachel: The question.

Researcher: Ok.

Rachel: Well, I could ask—will I catch up with her or not, but I don't know how to put the information.

At this point, all of Rachel's writing behaviors were oriented toward the culminating question, "Will I catch up with her?" She did eventually abandon the time and rate idea as a pertinent element in the problem (although she kept it in the problem for some unknown reason). The turning point for Rachel in her problem writing was when Bruce replied.

Figure 1: Rachel's Bus Stop Problem
to one of her exploratory remarks. (Bruce’s participation in the episode was uninitiated by either Rachel or me, and she responded to his assistance.)

Rachel: ... no, we both walked seven steps a minute and she stops every three minutes.

Bruce: To get a drink of water from the drinking fountain.

Rachel: To rest. And I’m—if I walk seven steps a minute, then I would get home in twelve minutes. (under breath) I don’t know what I’m doing, but ... would I catch her or not. Would that work? (laughs)

Bruce: How far away is her house?

Rachel: “We’re neighbors.”

It appeared, at this point, that she had finally identified her focal question. Later, Rachel asked a question similar to Bruce’s (“How far away is her house?”), but one that now provided her with the needed information to answer the question.

Rachel: I am—how far should I be away from my house?

Bruce: Eighty steps. Eighty steps away.

Rachel: Yeah! Good idea. No, it’s not. It has to be minutes. So, I-no-

Eventually, Rachel decided that she was 30 minutes away, her friend was 15 minutes away, she walked without stopping, and her friend stopped for 1 minute every 3 minutes. The item about time and rate (both girls walk 13 steps per minute) is extraneous information, but it was unlikely that Rachel was aware of this during the writing or her initial solution efforts. Finally, according to Rachel, she added the item about her friend being “thin” to give the problem more non-mathematical interest and extraneous information.

Jessica’s Problem. Jessica’s problem about report cards also illustrates question-directed writing behavior (see Figure 2).

Jessica: Ok, I think I’m gonna write one on report cards. (ten seconds later) Ok ... In each one of my grades I get 3 report cards including kindergarten.

Researcher: What are you thinking?

Jessica: See ... I’m thinking about how many reports have I gotten so far ... how many report cards am I gonna get? Or how—what am I gonna do for the problem part now because I’ve got mostly the story part—now I need to think of what am I going to do for the problem.

Jessica quickly generated a topic—a subject not unusual for fifth graders—and some unfocused information about the topic. She used this bit
In each one of my grades I get 3 report cards including kindergarten. I am in 5th grade. If I were in 8th grade, how many report cards would I have received?

Figure 2: Jessica’s Report Card Problem

of information to produce the beginning of a question. Her idea for a question involved the number of report cards over a still unclear amount of time. Subsequent writing focused on answering the question, still unwritten. The task was one of deciding the “period of time” condition.

Jessica: I’m thinking how—how high does grade school go? I think it’s like eighth grade.
Researcher: What are you thinking?
Jessica: (erases “In”) I’m thinking about just starting the question now because um... I could include part of it in my question.
Researcher: Part of what?
Jessica: Part of my story... ‘cause I’m gonna write the question in eighth grade, how many report cards have I received.
Researcher: Ok, ok.
Jessica: If I were in 8th grade how many report cards would I have received?

The question, how many report cards, led Jessica to generate the condition, if I were in eighth grade. She thought of a topic, wrote the first thing about report cards that came to mind (three a year), generated a question related to that bit of information (How many do I get?), and then added information (eighth grade) required to answer this question. She found it practical to frame the problem question and one of the problem conditions in the same sentence.

Bruce’s Problem. A very clear example of this type of goal-directed writing behavior is Bruce’s Arctic hiking problem (see Figure 3).

Before putting pencil to paper, Bruce considered a possible topic.

Bruce: Maybe I’ll start out with, um, I’m packing my stuff... even better than hiking, maybe I should... oh, I know...
Bruce: I'm gonna do about myself, how much stuff I need for hiking and how much money it's gonna cost.

Everything Bruce wrote was directed at the problem's solution: That is, how much is this trip going to cost? He drew upon his personal knowledge of ice hiking to write the problem. Once he determined the focal question, writing the text was routine.

Robbie's Problem. Robbie's problem is similar in its question's directedness (see Figure 4). Although the problem appeared routine, the composition of this problem was for him a labored process. Post-episode discussion suggested a goal-directed orientation.

Researcher: You knew that you were going to end up with this type of problem. Like, I have this much money. If I spent this and this and this . . .

Robbie: Un, huh.

Researcher: Oh, ok.

Robbie: Because that's how I think of a—I think of the question first. I think of the question first before I write it.

Later, Robbie continued:

Robbie: Yeah, and as soon as I took, I have thirty dollars, I knew I was gonna have, how much money do I have left?
I have $30.00. I want to buy a dropper, 10 test tubes, a boiling flask, and some rubber tubing. The eye dropper is 40 cents. Each $1.00. The boiling-flask is $5.00, and the rubber tubing is $1.00 a yard. I want 3 yards. How much money do I have left?

Figure 4: Robbie’s Chemistry Set Problem

Writing Behaviors That Focused on the Problem’s Topic

There were six episodes in which students did not generate the culminating question until after the problem’s text was completed. In these situations, the students used a free association strategy to write their problems: The problem-writer identified a topic and then wrote information related only to the topic itself rather than to any organizing question. The culminating question was identified only after the problem’s text was written. When this happened, there was a tendency for problem writers to write problems choosing topics about which they had limited background knowledge. The topics did not reflect personal experience or interest and, instead, were either impersonal or environmentally cued. Impersonal topics did not have any apparent relevance to anything in the students’ everyday experience (e.g., Eli’s “347 kids” problem described in the next section). Environmentally cued problems were prompted by an object in the environment, such as the tile pattern on the ceiling that Eli used as the basis of another problem.

By comparison, as noted in the previous section, there were 13 episodes in which students generated the culminating question early in the writing process. In only two of these episodes were the topics either impersonal or environmental. Story problems that were guided by culminating questions tended to be about topics wherein the problem writer had some background knowledge.

Beth’s Problem. Beth’s problem about a costume party is an example of a problem with an impersonal topic that was written without a guiding culminating question.

Beth generated an impersonal topic, a school costume party, and then created a hypothetical setting in the opening sentence. She then grappled briefly with the issue of a culminating question with these comments: “How can I make this into a problem,” and, later, “Um... I’m trying to think how I... put an end to the problem.” Unable to generate a question, she proceeded to write more information related to the opening line. (There are 10 guests...) The question she did write was related to the
general topic of the problem text and also resembled, in form, a familiar type of mathematics textbook problem (e.g., “How many ghosts, clowns, dogs, and cats will add up to . . . ?”). It appeared that Beth used her knowledge of story problem structure to write this problem, but she was confused about its meaning.

Robbie’s Problem. Robbie’s whale problem (his second) is an example of a problem topic that was interesting to the writer but one for which he did not have adequate background knowledge (see Figure 5). I observed Robbie writing this problem, and an excerpt from my field notes follows:

Robbie is reading a book about whales and he finds an interesting bit of info. He writes, “A blue whale [refers back to book] grows 200 pounds a day.” He goes back to reading, leafs pages, and tells (friend) that the skin is called blubber. Both boys laugh. He continues leafing through book, talking with (friend) about whales. Cuing into some new info, he continues writing. It also eats a thousand shrimp. At this point, he says, “Need more information.” He and friend leave room for library in search of other whale books.

When I met with Robbie again 10 days later, he showed me his completed problem. I asked him how he had generated the “7 years” information. He said that it was Jessica’s problem, shared a few days before during mathematician’s chair, that gave him the idea. Jessica’s problem read, I am eleven years old. How many days before I can vote? One step in the solution of Jessica’s problem required finding the difference between her current age (11) and the minimum voting age (18). Although the meaning of Jessica’s use of 7 years and Robbie’s incorporation of this information were different, it was useful to Robbie because it gave him an idea that led to the completion of his problem.

Strategies to Increase Problem Difficulty

I found evidence that students intentionally sought to increase the difficulty of their problems in 14 of 23 (61%) episodes. Problem writers attempted to make their problems more difficult in the following ways:

1. By using large or perceived to be difficult numbers (e.g., odd numbers were considered by students to be more difficult to manipulate than even numbers).
2. By adding extraneous, non-numerical information.
3. By adding extraneous, numerical information.
4. By adding pertinent (to the question) information.
5. By avoiding a standard or perceived to be routine problem question.
A blue whale grows 200 pounds a day. It also eats a thousand shrimp a day, it is 7 years old. How much does it weigh?

Figure 5: Robbie's Whale Problem

Some examples of these strategies follow.

Example 1: Eli made the key numerical information in one problem odd (instead of even) in order to increase difficulty.

Eli's story problem: If each kid in the school had 2 books how many books would there be if half of the kids put theirs together. There are 347 kids in the school and there are 34 teachers.

Researcher: You were first thinking about making it 500 kids, remember? And then you said, no, no, no. Three hundred and then... how come you went down—what were you thinking?

Eli: Because I wanted to make it an odd number.

Researcher: Why?

Eli: 'Cause it’s harder.

Example 2: Aaron chose to use a larger number in order to make the solution more difficult. The problem involves a visit to an auction with his dad; they had purchased two snowmobiles. The cost of fueling the snowmobiles was "$11.00 a piece."

Researcher: And later on you said that you wanted to make the problem harder. How come?

Aaron: Because just adding how much two snowmobiles were, you could have done that in a flash.

Researcher: What’s the point of making the problem harder? What were you thinking about?

Aaron: Makes it longer. If you add more things... (inaudible; but, meaning is that making problem longer makes it harder).

Researcher: Well, when you say make it harder, do you mean harder for you, harder for Mrs. Phillips or harder for the other kids?

Aaron: Harder for the other kids.

Researcher: Is that why you changed it to $11 a piece.

Aaron: Yeah.

Example 3: Bruce’s ambiguous story problem about a young tennis star included extraneous information that is both numerical and non-numerical. Bruce read a magazine article about this topic and wrote the problem at the same time.

Bruce’s problem: Jennifer Capriati could beat a bunch of 6-7 year olds at the age of 3, in tennis. She could swing a racket before she could walk said her dad. Now
at 13 she plays tennis in the 18 and under division while all the other 13 year olds play in the 14 and under division.

Jennifer is younger than most of her opponents and at 5'5" and 120 pounds she is also smaller and lighter. Her idol is Chris Evert who has won more tournaments than any one in history.

Jennifer has played in the wimbledon in England, French open, Itallin open, & American open this year. She plays twice a day every day with friends. She won a girl nation championship she practiced with Chris more than 15 times this year.

How many times has she played this year.

Early in the writing, Bruce clarified the type of story problem he was planning: "It's gonna be like a real story. I mean a biography . . . story problem." Throughout the episode, as he continued adding information, Bruce oscillated between worrying if the problem was too difficult or too easy:

"Ok, so . . . that would make it a little harder . . . And later, Oh, I don't want it to be too hard, but I want it to be kind of different and hard for them to do . . . so um, maybe I should put some more tournaments down because I'm in that paragraph and then do that information (inaudible)."

Just before he wrote the final question, he considered the possibility of writing more information. "Now I'll ask the question 'cause I've got all that done . . . maybe I should put more information . . . uh, it's too hard, ok . . ."

Example 4: For his problem about the weight of bears, Jeff struggled to write a culminating question that would be unusual and, therefore, more difficult.

Researcher: What are you thinking?

Jeff: Another word besides "average" . . . to say a different question besides just saying, what's the average weight? Instead of just saying, the average weight.

Researcher: Why?

Jeff: Well, then it makes it too easy I was thinking of do it . . . like, if you shot this bear and this bear, and then you got the meat and you put it in the freezer, and the freezer holds so many pounds . . . how many freezers would you need, but I don't think that's too easy-hard . . . well, maybe not—(laughs) I'm trying to think of another word to put instead of average . . . different question than instead of saying, what's the average weight . . .

For some time, the students had been writing "find the average" problems, so this genre of problem was quickly becoming routine for most of the students.

Example 5: The last strategy involved making a part of the problem non-obvious, thereby requiring an additional procedure for the solver. Notice the abrupt change in Jessica's thinking after her second sentence.
Jessica: Mrs. Phillips is my teacher. She can talk a mile a minute . . . There are—wait. School starts at 8:40 and ends at 3:20. She talks 314 of the day. How long do her pupils get to speak?

Later, Jessica told me that she was going to clearly state the length of the school day, but decided not to do this so that her peers would have to figure this out for themselves.

Discussion

The findings portray these fifth grade students as deliberate, thoughtful, and interesting mathematics story-problem writers. Implicitly (i.e., without being directed by instruction), the students appeared to have developed three strategies for writing mathematics story problems. The first strategy entailed the use of a focal culminating question that guided problem composition. The second was the use of free association writing, in which the writer generated information that was loosely related to the general problem topic and the culminating question was not identified until the problem text was completed. The third strategy involved a variety of techniques aimed at increasing the difficulty of problems.

Students who were able to define their culminating question early in the writing process tended also to be the high-status students in school mathematics. These students usually brought sense-making strategies to school mathematics. They tended to monitor their comprehension on problem-solving tasks and, when there were comprehension breakdowns, they implemented strategies to repair problems in a systematic manner. It makes sense that the more effective problem solvers in the class responded to this ill-defined problem-writing task (i.e., write a mathematics story problem about anything you want) by working to impose their own meaning. They did this, essentially, by asking: What is my question here? And, What exactly is it I am trying to discover?

In a minority of episodes, students did not use questions to guide their problem writing; and these students tended to be the low-status students in school mathematics. The behavior of these students was similar to a free association strategy typical of elementary students on school writing tasks (Scardamalia & Bereiter, 1985). After identifying the story problem topic, the students wrote several problem conditions that were loosely associated with that topic. These students did not identify one central, organizing idea or meaning to guide their composition. Interestingly, most of the problems in this category reflected topics that were impersonal; they did not relate to an actual experience or to an interest of the writer. This might be expected because actual personal experiences and interests often naturally suggest questions and problems. When the topic
reflects real experience or some developed background knowledge, the problem writer only has to reflect upon that lived experience or knowledge in order to generate the culminating question. For example, Robbie's topic about a chemistry set contained the lived experience of paying for the set, receiving change, and so on. His think aloud indicated the use of a guiding focal question. But students, when their topics did not reflect real experience or interests, tended not to use focal questions.

This is not to suggest that unfocused story problems, with ambiguous meaning, do not present educational possibilities. Robbie's whale problem, although not question-driven, suggested several learning opportunities. First, the problem gave him an opportunity to organize some new information about a topic of interest, whales. Second, when he attempted to solve it later, the solution did pose difficulties and, therefore, provided an opportunity for the learning of arithmetic procedures. Third, his problem reflected a misconception about the growth rate of whales. This misconception presented another learning opportunity, particularly as he shared the problem with skeptical peers and adults.

A free association writing strategy may be useful for story problem writers when their mathematics problem or question is unclear. Here, the purpose of early writing simply is to generate information that, in some way, involves mathematics (e.g., a trip to the supermarket, redesigning one's bedroom, trading sports cards, betting). Once the student has generated this information, perhaps in a rough draft of a mathematics story or narrative, raising questions from this writing may be less demanding than doing it in "their heads." Associational writing as a strategy to generate information related to everyday mathematics experience is an example of how writing might support learning in mathematics. However, without the expectation that students employ their associational writing as a vehicle for problem posing and continued reflection upon a particular mathematics experience, this type of writing may not effectively serve students' mathematics learning.

The findings also demonstrate students' resourcefulness in developing strategies to make their problems more difficult. A motive for these strategies appeared to be to write problems that peers would then have difficulty understanding or solving. The students seemed to enjoy the challenge of writing difficult problems for their peers because difficult problems were held in high esteem by classmates (Winograd, 1991b). However, for the present study, it appeared that students' motivation to write difficult problems for peers sometimes dominated their thinking during problem writing. In addition, although topics tended to reflect actual experiences and interests, the questions posed were somewhat contrived and artificial. The self-consciousness demonstrated in the use of these strategies may have diminished the opportunity for students to
use mathematics stories and story problems more effectively as a vehicle to better understand and make sense of mathematics applications in their everyday lives. Still, another perspective suggests that the pleasure students felt from the challenge of writing difficult problems (particularly for their peers' problem-solving work) is a legitimate use of mathematics. Bishop (1988) identified "play" as one everyday activity, common to all cultures, from which some aspects of formal mathematics as a discipline have evolved. The problem-writing activity did appeal to the students' natural interest in play and games, activities often of a competitive nature. I believe this context of "mathematics problem writing as play" can serve as one sensible point of departure for mathematics teaching and learning.

The kinds of behaviors exhibited by most students in this study are similar to the goal-directed behavior of effective writers described by Flower and Hayes (1980). The students purposefully worked to write problems that were interesting and difficult, particularly for what they perceived to be an interested peer audience. Many students, especially those of higher status, also tried to identify a central main idea for their problems in the form of culminating questions. Interestingly, writing teachers typically encourage the use of controlling, or governing, questions to guide the composition of expository text. Ede (1992), for example, recommended that one planning strategy for expository writers is to analyze the rhetorical situation; this analysis entails writers asking questions about their intention for the piece, goals for affecting the reader, and the constraints of the textual form. If mathematics story problems can be viewed as one type of expository prose, the fifth-grade students' efforts to raise controlling questions during story problem writing is fundamentally no different than what effective writers do when they write reports, letters, or essays.

The students' responses were self-initiated without prompting by the classroom teacher or researcher. The inducement for these students' planning behaviors may reflect several prevailing beliefs among the students (Winograd, 1991b). For example, students expressed a belief that the "good" mathematics student enjoyed writing story problems; that the "good" story problem was difficult; and that the difficult problem contained non-mathematical content that reflects everyday experience. These beliefs were articulated by students when at the end of mathematician's chair, the problem writer asked the audience, "What did you like about my problem?" In response to this expression of appropriate behavior (a definition that took shape during the first six weeks of the research), I sensed that students tried to write challenging and interesting problems because, in part, these types of problems were held in high esteem by students in this classroom.
Implications for Teaching

The data describe 8 students who were willing and able to write problems that were moderately challenging (to the writer or to others in the class), that contained viable mathematics concepts and skills, and that were embedded in everyday experience or imagination. But certain instructional conditions may need to be in place for this to occur with any regularity with these students and, perhaps, with students in other settings. Adapting Cambourne's (1988) conditions for oral language development, there are eight conditions that may support students' development as problem writers in mathematics.

1. Students should be exposed to models of the kinds of problems that we expect them to write. Examples of these problems, as well as topics for problems, should come from the teacher, the students, and the formal curriculum. If students are to write probability and statistics problems, teachers should first organize classroom experiences that evoke a need for this use of mathematics and then immerse students in these types of problems. If students are to reflect with more elaboration upon their everyday mathematics experiences, perhaps in the form of personal narrative, teachers must first immerse students in this type of mathematics writing. (It is from this genre of "personal mathematics narrative" that more interesting and honest culminating questions and problems may emerge.) Finally, problems written by teachers and students should reflect a wider definition of mathematics, beyond the arithmetic that dominates many elementary classrooms, to include the full range of topics contained in the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989): statistics, probability, geometry, measurement, estimation, and process problems. Included, as well, should be problems that are purely conceptual and not directly related to everyday mathematics applications (e.g., Brown & Walter, 1983).

2. Students require explicit demonstrations from teachers regarding how to identify topics for problems and how to use those topics to write problems. Teachers need to model how they use mathematics to make sense of their lives, how mathematics is a tool that can be used to solve real problems, or to solve problems that reflect imagination and creativity. Students' everyday lives certainly are rich in mathematical experience, and the topics and questions of their problems would clearly be more authentic if students were more aware of how mathematics relates to their experiences. Calkins (1991) described how elementary students kept notebooks outside school, not unlike the notebooks that writers or anthropologists in the field might keep. When students become aware of an important
personal experience or idea, they make a note of it and then use it later as the basis for a story or project. Elementary students can be encouraged to keep notebooks that contain notes about everyday mathematics experience, problems, and questions. Of course, teachers need to model this by also keeping journals of this kind and then sharing them with their students.

3. Students need to retain control over the mathematics and non-mathematics content of their problems. Student-centered control increases the likelihood that students will engage in problem writing and problem solving in a more wholehearted manner. Inevitably, individual students will write problems that do not meet curricular goals, especially in the case of students writing routine and formulaic word exercises. If not from their own problems, these students can still have educative problem-solving experiences, but the problems will be those written by peers or the teacher, or contained in curricular materials.

4. Students must practice mathematics problem solving every day. And almost as often, they need to write, solve, and then share their own problems. Problem writing and problem solving should become a predictable part of the school mathematics experience. The regularity of this activity may influence students to initiate similar mathematics problem-writing and problem-solving activity on their own, outside school.

5. Students profit from regular feedback from peers and from their teacher about the meaning of their problems and solutions. Partnering, small groups of three or four, and even whole class discussion provide students with a forum for feedback. While students respond differently to varying configurations of audience, the predictability of having some audience for students’ self-generated problems appears to be a central motivation for problem writers to write interesting and challenging problems.

6. Students require the conceptual and strategic knowledge to understand and solve mathematics problems. I have observed students become discouraged as problem writers when they did not know how (or did not have the confidence) to solve or understand their problems, or they did not have the discussion skills to explain their problems to peers. Problem writing as a core component of the school mathematics program can lead to students’ learning only when it is coupled with a well-articulated curriculum for teaching mathematics concepts and problem solving. An important rationale for having students write their own mathematics problems is that it stimulates a "friendly" context for teacher instruction. The task for teachers is to guide students toward the writing of certain desirable types of problems (i.e., ones that meet curricular goals and that will
promote mathematics learning) and then use students' difficulties with those problems as one basis for instruction. In a mathematics problem-writing and problem-solving classroom, teachers have a great deal to teach.

7. Students need the time to develop as problem writers, problem solvers, and discussants of mathematics. Teachers also need the time to develop this approach to mathematics teaching. It is not unusual for students to write bland, formulaic "word exercises" similar to what is typically found in the worst textbooks if the classroom conditions are not right. And, it is not unusual for students to write problems which are too difficult to solve or understand, even with the help of an adult. I do not think classroom conditions may ever be exactly right for all students at all times. Graves (1983) noted variability in the quality of students' writing from one piece to the next; students' mathematics problems will also reflect this variability. For example, students may write a problem richly embedded in experience with elements of non-routine mathematics one day, and during the next three days, they may write simplistic word exercises.

A broader view of mathematics and how it is learned entails a wide range of classroom experiences: students writing, solving, and sharing original problems; students solving and sharing their solutions of adult-generated problems; teachers providing instruction in mathematics concepts, problem solving, problem writing, and discussion strategies; teachers posing problems that may take several days to solve; and students using various technologies, such as calculators and computers, to learn and do mathematics. This broader conception of mathematical literacy suggests the need for an extended mathematics period as well as a more integrated approach to curriculum. Certainly, the time usually allocated in elementary classrooms to mathematics needs to be extended, perhaps to at least 75 minutes per day.

8. Students' accountability for their performance in mathematics should be the same whether the problems derive from textual sources, the teacher, or from the students themselves. Student accountability here includes writing and solving self-generated problems, preparing to teach or to defend problems to peers, and working problems written by peers. In spending the time necessary to incorporate a student-centered, problem-writing program in mathematics, teachers are communicating to students that their problems are important and valued.

A tentative model of a mathematics-story problem writing process integrates the question-directed and free association writing strategies employed by the students in Kathy's classroom. First, the
teacher identifies the conceptual or strategic knowledge the students are to learn. The teacher then facilitates some relevant instructional experiences, including the immersion of students in problems reflecting this knowledge. Students who already have the appropriate background knowledge need only cueing to stimulate conscious awareness of this knowledge. If the mathematics content can be applied to some everyday experience, the teacher helps students identify real-life situations that reflect this goal. For example, decimals suggests the everyday situation of money and shopping. Probability suggests situations like weather forecasting, betting, and games of chance. If the content is more abstract, such as set theory, then students should be guided to write similarly abstract questions and problems that reflect this content.

After some exploratory reflection on these mathematical situations (e.g., by the use of free writing, discussion, or drawing), students are more likely to have adequate background knowledge to identify sensible culminating questions. They then write the problem text, making sure it contains enough information to answer their questions. Students work to solve their problems, share these problems and solutions with peers, and then revise problems and solutions based on peer and teacher feedback. Finally, students "publish" their problems on worksheets, in anthologies, or in some other format, perhaps for an audience of classmates or students from other classes.

This model for problem writing is similar to models of the writing process and writers' workshop. Linkage of the writing process, workshop approach to mathematics problem writing became quite clear to me while reading Writing: Teachers and Students at Work (Graves 1983). If I substituted "story problem" every time I read the words, "story" and "piece," it made sense for mathematics learning.

Limitations of the Present Study and Recommendations for Future Research

The study has several limitations, and these limitations suggest many opportunities for future research and teaching. First, the findings reflect my experience with 8 students. The ideas contained in the Implications section should be tested, criticized, and refined in ways that reflect the realities of other students, teachers, and settings. More in-depth, qualitative case studies are needed, studies that carefully document students' mathematics behavior and learning over a long period of time in socially interactive, problem-writing classrooms.
Threats to reliability in my interpretation of the data were reduced in two ways (Goetz & LeCompte, 1984). First, I shared the findings and illustrative data with the teacher and some students. They acknowledged that my interpretation of the data made sense to them. Second, I have included an ample amount of primary data in this report, thereby permitting an adequate review and evaluation by the reader. However, a formally established internal reliability index was not established.

This research did not address the link between problem writing and problem solving. Some questions about that linkage remain. If students have regular opportunities to write and solve their own mathematics problems, particularly in the type of classroom environment described in the previous section, how will problem writing influence the learning of mathematics? Will students be more receptive toward adult instruction when the instruction relates directly to the content of their problems?

This study did not examine the mathematics content of students' problems, nor did the research systematically attempt to influence the mathematics content of problems. What kinds of instructional experiences will lead students to write problems that reflect a broader view of mathematics (National Council of Teachers of Mathematics, 1989), while at the same time enabling students to maintain control of the content of their problems?

In the present research, students were not provided with a structure or procedure to guide their composition of story problems. Can students be taught to engage in more deliberate, planning behaviors when writing problems, such as identifying a culminating question early in the writing process? How can we help students become more aware of the mathematics-related questions and problems contained in everyday life? Can students be taught to observe and study the mathematics in their everyday lives, perhaps by using a writer's notebook (Calkins, 1991), and then use these topics as a source for more authentic story problems?

Finally, further research should examine how and why students go about writing problems that they then have difficulty understanding or solving. How can teachers lead students to write problems that are moderately challenging? In what ways does the presence of an interested or critical peer audience influence the behavior of problem writers?

I believe the composition of mathematics story problems is a fertile area for research and one that can effectively link innovations in the teaching of writing with the current interest in applications for writing in the mathematics curriculum.

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